NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	LEVEL: 7
COURSE CODE: DAE702S	COURSE NAME: DESIGN AND ANALYSIS OF EXPERIMENTS
SESSION: JANUARY 2020	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER	
EXAMINERS	DR C.R KIKAWA
MODERATOR:	PROF PETER NJUHO

INSTRUCTIONS	
	1. Answer ALL the questions in the booklet provided.
	2. Show clearly all the steps used in the calculations.
	3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

ATTACHMENTS

Standard Normal Distribution table, Inverse Cum Freq Distribution table, t-table, f-tables
(T-12 to T 19)

THIS QUESTION PAPER CONSISTS OF 6 PAGES (including this front page)

NAMIBIA UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS AND STATISTICS
DESIGN & ANALYSIS OF EXPERIMENTS: DAE702S

SECOND/SUPPLEMENTARY EXAMINATION: JANUARY 2020
Time-3 Hrs   Attempt all Questions   Maximum Marks - 100

1. Question

   (a) Discuss the following concepts as used in experimental design:

      1. Treatment
      2. Experimental units
      3. Responses
      4. Randomization
      5. Confounding

                                                      (15 marks, (3@))

   (b) Briefly discuss two examples that could best distinguish an experimental unit and a measurement unit.
       (**Hint:** One example should be agricultural and the other educational)   (5 marks)

2. Question
   The viscosity of a liquid detergent is supposed to average 800 units at $25^{o}c$. A random sample of 16 batches of detergent is collected, and the average viscosity is 812 units. Suppose we know that the standard deviation of viscosity is $\sigma = 25$ units.

   (a) State the hypotheses that should be tested.   (2 marks)

   (b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?   (10 marks)

   (c) What is the p-value for the test in part (b)?   (3 marks)

   (d) Find a 95% confidence interval on the mean.   (5 marks)

3. Question
   The tensile strength of portland cement is being studied. Four different mixing techniques can be used economically. The data in Table 1 have been collected

Table 1: Results of Tensile strength and Mixing Techniques

Mixing Technique	Tensile Strength			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

   (a) Construct an appropriate ANOVA table. (13 marks)

   (b) Test the hypothesis that mixing techniques affect the strength of the cement. Use $\alpha = 0.05$. What are your conclusions? (7 marks)

4. Question
   An experiment is conducted to compare four different mixtures of the components oxidizer, binder, and fuel used in the manufacturing of rocket propellant. To compare the four mixtures, five different samples of propellant are prepared from each mixture and readied for testing. Each of five investigators is randomly assigned one sample of each of the four mixtures and asked to measure the propellant thrust. These data are summarized in the table below. Use $\alpha = 0.05$.

Mixture	Investigator				
	1	2	3	4	5
1	2,340	2,355	2,362	2,350	2,348
2	2,658	2,650	2,665	2,640	2,653
3	2,449	2,458	2,432	2,437	2,445
4	2,403	2,410	2,418	2,397	2,405

Figure 1: Table of values

   (a) Construct a general ANOVA display for the randomized complete block design (RCBD), clearly defining all variables being used. (No calculation required). (8 marks)

   (b) Suppose that there are a treatments (factor levels) and b blocks. Write an effects model for the RCBD. (5 marks)

   (c) State the appropriate hypotheses for the experiment. (2 marks)

(d) Using the information in the output given in the ANOVA table below, test the hypotheses stated in part (c). What are your conclusions? (5 marks)

Tests of Between-Subjects Effects

Dependent Variable: Thrust

Source	df	Type III Sum of Squares	Mean Square	F	Sig.
treatment	3	261260.9500	87086.9833	1264.7269	.0000
block	4	452.5000	113.1250	1.6429	.2273
Error	12	826.3000	68.8583		
Corrected Total	19	262539.7500			

Figure 2: General ANOVA table for a RCBD

5. Question

Here we quote an experiment that had been designed as a Latin square. The skins of rabbits' backs were inoculated with a diffusing factor in six separate sites. Six rabbits were therefore used and the order in which the sites were inoculated was done six different ways. The outcome measured was area of blister (cm). The overall objective was to see whether or not the order of administration affected this outcome. The experimental design and data are represented in the Latin square below.

		Rabbit					
		1	2	3	4	5	6
	a	iii	v	iv	i	vi	ii
		7.9	8.7	7.4	7.4	7.1	8.2
	b	iv	ii	vi	v	iii	i
		6.1	8.2	7.7	7.1	8.1	5.9
	c	i	iii	v	vi	ii	iv
		7.5	8.1	6	6.4	6.2	7.5
Position	d	vi	i	iii	ii	iv	v
		6.9	8.5	6.8	7.7	8.5	8.5
	e	ii	iv	i	iii	v	vi
		6.7	9.9	7.3	6.4	6.4	7.3
	f	v	vi	ii	iv	i	iii
		7.3	8.3	7.3	5.8	6.4	7.7

Figure 3: Latin Square Design and Data

(a) You are required to construct a test workbook (ANOVA worksheet: Observations; Rabbit; Position; Order). **(8 marks)**

4

(b) From the Latin square test presented below, discuss the research findings (Strictly stick to the objective of the research as stated). **(5 marks)**

Factors: Rabbit, Position, Order.

Source of Variation	Sum Squares	DF	Mean Squa
Rows	3.833333	5	0.766667
Columns	12.833333	5	2.566667
Treatments	0.563333	5	0.112667
Residual	13.13	20	0.6565
Total	30.36	35	

F (rows) = 1.167809, P = .3592

F (columns) = 3.909622, P = .0124

F (treatments) = 0.171617, P = .9701

Figure 4: Results table for the Latin square design

(c) Discuss the concepts of a Latin Square design. **(7 marks)**

END

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Table 5 Normal distribution — inverse cumulative distribution function

0.50	0.0000	0.60	0.2533	0.70	0.5244	0.80	0.8416	0.90	1.2816	0.99	2.3263
0.51	0.0251	0.61	0.2793	0.71	0.5534	0.81	0.8779	0.91	1.3408	0.991	2.3656
0.52	0.0502	0.62	0.3055	0.72	0.5828	0.82	0.9154	0.92	1.4051	0.992	2.4089
0.53	0.0753	0.63	0.3319	0.73	0.6128	0.83	0.9542	0.93	1.4758	0.993	2.4573
0.54	0.1004	0.64	0.3585	0.74	0.6433	0.84	0.9945	0.94	1.5548	0.994	2.5121
0.55	0.1257	0.65	0.3853	0.75	0.6745	0.85	1.0364	0.95	1.6449	0.995	2.5758
0.56	0.1510	0.66	0.4125	0.76	0.7063	0.86	1.0803	0.96	1.7507	0.996	2.6521
0.57	0.1764	0.67	0.4399	0.77	0.7388	0.87	1.1264	0.97	1.8808	0.997	2.7478
0.58	0.2019	0.68	0.4677	0.78	0.7722	0.88	1.1750	0.975	1.9600	0.998	2.8782
0.59	0.2275	0.69	0.4958	0.79	0.8064	0.89	1.2265	0.98	2.0537	0.999	3.0902

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
						Confidence Level					
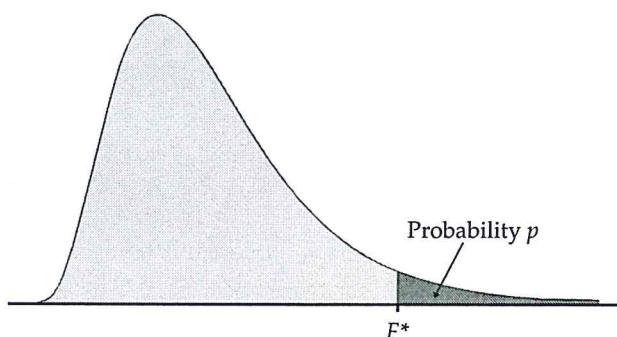
Table entry for p is the critical value F^* with probability p lying to its right.

Probability p

F^*

TABLE E

F critical values

	p	Degrees of freedom in the numerator								
		1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33

Degrees of freedom in the denominator

TABLE E

F critical values (continued)

			Degrees of freedom in the numerator								
	p	1	2	3	4	5	6	7	8	9
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75

Degrees of freedom in the denominator

TABLE E

F critical values (continued)

			Degrees of freedom in the numerator								
	p	1	2	3	4	5	6	7	8	9	
18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	
	.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	
	.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	
	.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	
	.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	
19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	
	.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	
	.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	
	.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	
	.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	
20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	
	.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	
	.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	
	.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	
	.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	
21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	
	.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	
	.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	
	.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	
	.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	
22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	
	.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	
	.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	
	.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	
	.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	
23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	
	.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
	.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	
	.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	
	.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	
24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	
	.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
	.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	
	.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	
	.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	
25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	
	.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	
	.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	
	.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	
	.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	
26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	
	.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	
	.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	
	.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	
27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	
	.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	
	.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	
	.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	

Degrees of freedom in the denominator

TABLE E

F critical values (continued)

| | | p | Degrees of freedom in the numerator |
			1	2	3	4	5	6	7	8	9
Degrees of freedom in the denominator	28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
		.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
		.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
		.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
		.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
	29	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
		.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
		.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
		.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
		.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
	30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
		.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
		.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
		.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
		.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
	40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
		.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
		.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
		.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
		.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
	50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
		.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
		.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
		.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
		.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
	60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
		.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
		.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
		.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
		.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
	100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
		.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
		.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
		.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
		.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
	200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
		.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
		.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
		.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
		.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
	1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
		.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
		.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
		.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
		.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13